

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

Int 3MA C12 DDAY

Name _____ Per _____ Group _____

1. Simplify these fundamental identities

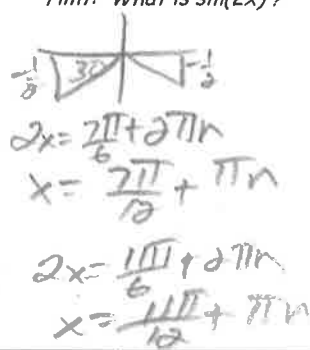
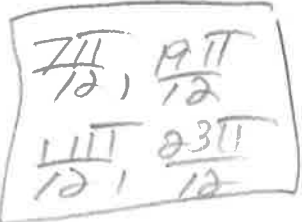
a. $\sin^2 x - 1 = -\cos^2 x$	b. $1 + \tan^2 x = \sec^2 x$	c. $\frac{1}{\csc \theta} = \sin \theta$	d. $\cos(-x) = \cos x$
e. $\csc^2 x - 1 = \cot^2 x$	f. $\frac{\cos x}{\sin x} = \cot x$	g. $1 - \cos^2 x = \sin^2 x$	h. $\csc^2 x - \cot^2 x = 1$

2. Solve $4 \sin x \cos x = -1$ for the interval $[0, 2\pi)$

Hint: What is $\sin(2x)$?

$$2(2 \sin x \cos x) = -1$$

$$\sin 2x = -\frac{1}{2}$$



3. Solve $\cos x + 2 = 2 \sin^2 x$ for all x

$$\cos x + 2 = 2(1 - \cos^2 x)$$

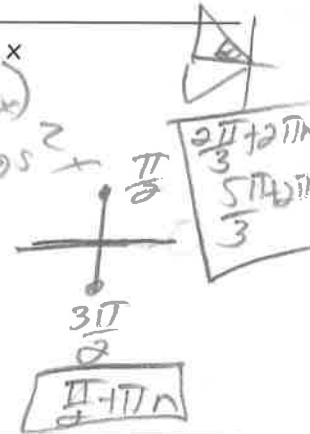
$$\cos x + 2 = 2 - 2 \cos^2 x$$

$$2 \cos^2 x + \cos x - 0 = 0$$

$$\cos x(2 \cos x + 1) = 0$$

$$\cos x = 0 \quad 2 \cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

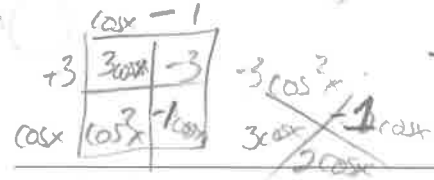


3. Find all solutions in the interval $[0, 2\pi)$:

$$\cos^2 x + 2 \cos x - 3 = 0$$

$$(\cos x + 3)(\cos x - 1) = 0$$

$$\cos x = -3 \quad \cos x = 1$$



4. Find all solutions in the interval $[0, 2\pi)$:

$$\sin 2x = \sin x$$

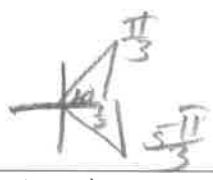
$$2 \sin x \cos x = \sin x$$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x(2 \cos x - 1) = 0$$

$$\sin x = 0 \quad 2 \cos x - 1 = 0$$

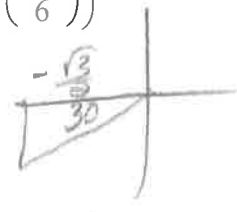
$$\pi, 0 \quad \cos x = \frac{1}{2}$$



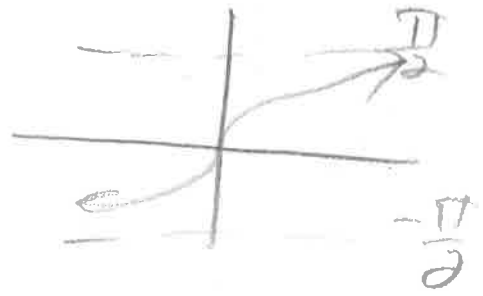
5. Find the exact value $\cos^{-1}(0)$



6. Find the exact value $\sin^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$



7. Sketch $y = \tan^{-1} x$ and show domain & range.



Domain:

Range:

8. Find $\cos 2A$ if $\sin A = \frac{5}{13}$ and A is in the second quadrant.

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$



$$\frac{144}{169} - \frac{25}{169}$$

$$5^2 + x^2 = 13^2$$

$$\sqrt{x^2} = \sqrt{144}$$

$$\frac{119}{169}$$

9. Verify $\frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

$$\frac{\cancel{\tan x} + \cancel{\cot y}}{\cancel{\tan x} \cancel{\cot y}} = \frac{1}{\cot y} + \frac{1}{\tan x}$$

$$= \tan y + \cot x$$

10. Verify $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2 \csc^2 x$

$$\frac{1+\cos x}{1+\cos x} \cdot \frac{1}{1-\cos x} + \frac{1}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}$$

$$= \frac{1+\cos x + 1-\cos x}{1-\cos^2 x}$$

$$= \frac{2}{\sin^2 x} = 2 \csc^2 x$$

11. Write the equation of the parabola $y = ax^2 + bx + c$ passing through points $(0, -7), (-1, -8)$ and $(2, 13)$.

$$-7 = a(0)^2 + b(0) + c$$

$$-7 = c$$

$$-8 = a - b - 7$$

$$-1 = a - b$$

$$10 = 2a + b$$

$$\frac{9 = 3a}{3 = a}$$

$$13 = 4a + 2b - 7$$

$$20 = 4a + 2b$$

$$10 = 2a + b$$

$$10 = 2(3) + b$$

$$10 = 6 + b$$

$$4 = b$$

$$y = 3x^2 + 4x - 7$$

12. Solve $\begin{cases} x = 2 - y - z \\ 2x + z = 7 \\ x - y + 3z = -4 \end{cases}$

$$\begin{aligned} (A) \quad & x + y + z = 2 \\ (B) \quad & 2x - y + 3z = -4 \\ (A+B) \quad & 3x + 4z = -2 \end{aligned}$$

$$\begin{aligned} 2x + z &= 7 \\ x - y + 3z &= -4 \end{aligned}$$

$$\begin{aligned} 2x + 4z &= -2 \\ -2x - z &= -7 \end{aligned}$$

$$3z = -9$$

$$z = -3$$

$$5 = 2 - y + 3$$

$$0 = -y$$

$$0 = y$$

$$2x + 3 = 7$$

$$2x = 4$$

$$x = 2$$

$$(2, 0, -3)$$

14. Solve $15 - 2(7^{3x+4}) = 8$

$$-2(7^{3x+4}) = -7$$

$$7^{3x+4} = 3.5$$

$$\log_7 3.5 = 3x + 4$$

$$1.6451 = 3x + 4$$

$$-1.1181 = 3x$$

$$-0.3727 = x$$

15. Find the sum of the sequence: $64 + 16 + 4 + \dots$

$$S_{\infty} = \frac{64}{1 - \frac{1}{4}} = \frac{64}{\frac{3}{4}} = \frac{256}{3} \approx 85.33$$

$$64 \cdot r = 16$$

$$r = \frac{1}{4}$$

16. Solve $\frac{2x}{x+5} - \frac{1}{4} = x$

$$\frac{2x}{x+5} - \frac{1}{4} = x$$

$$\frac{8x - (x+5)}{4(x+5)} = x$$

$$8x - x - 5 = 4x(x+5)$$

$$7x - 5 = 4x^2 + 20x$$

$$0 = 4x^2 + 13x + 5$$

$$x = \frac{-13 \pm \sqrt{13^2 - 4(4)(5)}}{2(4)} = \frac{-13 \pm \sqrt{89}}{8}$$

17. Find the value of the third term of the binomial expansion of $\left(\frac{3}{2x^2} - \frac{x^3}{y^4}\right)^8$

$$\text{expansion of } \left(\frac{3}{2x^2} - \frac{x^3}{y^4}\right)^8$$

$${}^8C_2 \left(\frac{3}{2x^2}\right)^6 \left(-\frac{x^3}{y^4}\right)^2$$

$$28 \left(\frac{729}{64x^{12}}\right) \left(\frac{x^6}{y^8}\right)$$

$$\frac{5103}{16x^6 y^8}$$

18. Consider the expansion of $\left(2x^3 + \frac{3}{x}\right)^8 = 256x^{24} + 3072x^{20} + \dots + kx^0 + \dots$ Find k.

$${}^8C_6 (2x^3)^2 \left(\frac{3}{x}\right)^6$$

$$28 (4x^6) \left(\frac{729}{x^6}\right) = 81648$$