Int 3MA C10 DDAY Sequences & Series Name: Key Period: ___ Group #___

1. Find the sum algebraically $\sum_{i=5}^{13}(200(0.8)^{n-1})$

$$S_9 = \frac{81.92(1-.8^9)}{(1-.8)}$$

$$\approx \boxed{354.6244}$$

2. Find the sum of the sequence: $\frac{3}{4}-\frac{6}{12}+\frac{12}{36}-\ldots$

$$r = -\frac{2}{3}$$

$$S = \frac{3/4(1-\frac{2}{3}^{\infty \to 0})}{1-(-\frac{2}{3})} \to \frac{3/4}{5/3}$$

$$\frac{3}{4}\cdot\frac{3}{5} \to \boxed{\frac{9}{20}} \text{ OR } 0.45$$

3. Find n if $S_n = -1458$ for $\sum_{n=1}^{\infty}(15-3n)$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$-1458 = \frac{n}{2}(12 + 15 - 3n)$$

$$-2916 = n(27-3n)$$

$$0 = 3n^2 - 27n - 2916$$

$$0 = n^2 - 9n - 972$$

$$n = \frac{9 \pm \sqrt{81-4(-972)}}{2} \to \frac{9 \pm 63}{2} \to \boxed{36}, \cancel{-27}$$

4. Given the series $2368+592+148+\ldots$ find n if $a_n = 2.3125$

$$r = \frac{1}{4}$$

$$a_n = a_1 \cdot r^{n-1}$$

$$2.3125 = 2368\left(\frac{1}{4}\right)^{n-1}$$

$$.0009765625 = .25^{n-1}$$

$$\log_{.25}(.0009765625) = n-1$$

$$\boxed{n = 6}$$

5. Find a_n algebraically for the arithmetic sequence if $a_{10} = 282$, $a_{15} = 414$

$$a_{15} = a_{10} + (15-10)d$$

$$414 = 282 + 5d$$

$$d = 26.4$$

$$a_{10} = a_1 + 9(d)$$

$$282 = a_1 + 237.6$$

$$a_1 = 44.4$$

$$\boxed{a_n = 44.4 + (n-1)(26.4)}$$

6. Find the sum of the first 42 terms of $43+60+77+\ldots$

$$S_{42} = \frac{42}{2}(43 + a_{42})$$

$$a_{42} = 43 + (42-1)(17) \to 740$$

$$S_{42} = 21(43+740)$$

$$S_{42} = \boxed{16{,}443}$$

7. Find the sum: $1192+1181+1170+\ldots+389$

$$d = -11$$

$$a_n = a_1 + (n-1)d$$

$$389 = 1192 + (n-1)(-11)$$

$$\to n = 74$$

$$S_{74} = \frac{74}{2}(1192+389)$$

$$= \boxed{58{,}497}$$

8. Evaluate algebraically $\sum_{n=-3}^{11}(7n-4)$

$n = 11-(-3)+1 \to 15$ terms

1st term: $7(-3)-4 = -21-4 \to -25$

Last term: $7(11)-4 = 77-4 \to 73$

$$S_{15} = \frac{15}{2}(-25+73)$$

$$= \boxed{360}$$

9. Prove by induction: $-32+-12+8+\ldots+(20n-52)=\frac{n(20n-84)}{2}$

Prove true for $n=1$ $\quad -32 = \frac{1(20-84)}{2} \rightarrow -32 = \frac{-64}{2} \rightarrow -32=-32$ ✓ ☺

Assume true for $n=k$ $\quad -32+-12+8+\ldots+20k-52 = \frac{k(20k-84)}{2}$

Prove true for $n=k+1$ $\quad \underbrace{-32+-12+8+\ldots+(20k-52)}+20(k+1)-52 = \frac{(k+1)(20(k+1)-84)}{2}$

$$\frac{k(20k-84)}{2} + \frac{2(20k-32)}{2} =$$

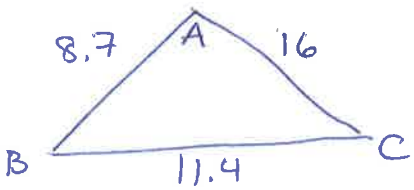
$$\frac{20k^2-84k+40k-64}{2} =$$

$$\frac{20k^2-44k-64}{2} =$$

$$\frac{(k+1)[20(k+1)-84]}{2} = \frac{(k+1)(20(k+1)-84)}{2}$$

QED

10. Calculate the angle A of ΔABC given $a=11.4, b=16, c=8.7$

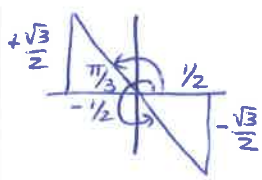


$11.4^2 = 8.7^2+16^2-2(8.7)(16)\cos A$

$\boxed{A \approx 43.56^\circ}$

11. Solve for all angles within $[0,2\pi]$. Show work.

$\cot\theta = -\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{-1/2}{\sqrt{3}/2} \quad \tan\theta = \frac{-\sqrt{3}/2}{1/2}$



$\frac{2\pi}{3}$ $\quad$ $\frac{5\pi}{3}$

12. Solve all roots of $y=x^3-5x^2-17x+21$

$\frac{P}{Q} = \pm1 \ \pm3 \ \pm7 \ \pm21$

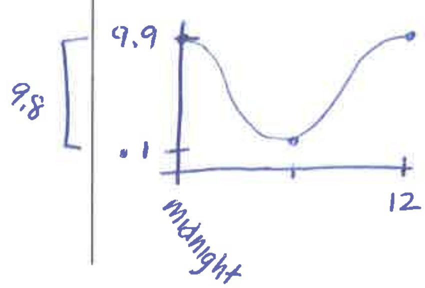
$$\begin{array}{r|rrrr} 1 & 1 & -5 & -17 & 21 \\ & \downarrow & 1 & -4 & -21 \\ \hline & 1 & -4 & -21 & ☺ \end{array} \quad \boxed{x=1}$$

$x^2-4x-21=0$

$(x-7)(x+3)=0$

$\boxed{x=7}$ $\boxed{x=-3}$

13. On February 10, 1990, high tide in Boston was at midnight. The water level at high tide was 9.9 feet; later, at low tide, it was 0.1 feet. Assuming the next high tide is exactly 12 hours later and that the height of the water is given by a sine or cosine curve, find a formula for water level in Boston as a function of time.



$y = 4.9\cos\frac{2\pi}{12}x + 5$