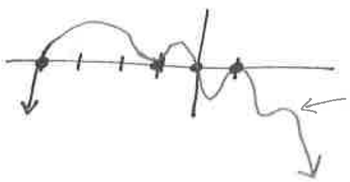
Int 3MA C8 DDAY Name Key Per ____ Group ____

1. Sketch the graph $y=-x(x+4)(x^2-1)^2(x^2+2)$

$x=0,-4,\pm 1$ bounce, $\cancel{\pm\sqrt{2}}$ i $\boxed{\downarrow\downarrow}$



2 other complex roots (somewhere)

2. Write as the product of linear factors

$y=(x^2-9)^2(x^4-4)$

$y=(x-3)^2(x+3)^2(x^2-2)(x^2+2)$

$\boxed{y=(x-3)^2(x+3)^2(x-\sqrt{2})(x+\sqrt{2})(x+i\sqrt{2})(x-i\sqrt{2})}$

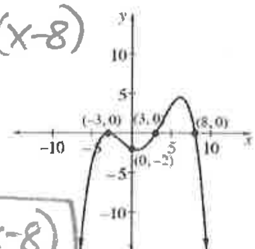
3. Write the exact equation of the polynomial function

$f(x)=a(x+3)^2(x-3)(x-8)$

$-2=a(9)(-3)(-8)$

$a=-\frac{1}{108}$

$\boxed{f(x)=-\frac{1}{108}(x+3)^2(x-3)(x-8)}$



4. Write the exact equation of the polynomial function

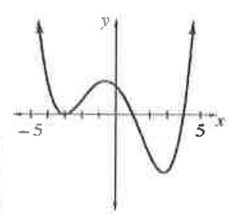
$f(x)=a(x+3)^2(x-1)(x-4)$

$3=a(9)(-1)(-4)$

$a=\frac{1}{12}$

$\boxed{f(x)=\frac{1}{12}(x+3)^2(x-1)(x-4)}$

y-intercept: (0, 3)



5. Write a polynomial function in standard form with the given roots. $x=-1,0,\frac{2}{3},\frac{1}{4}$

	$12x^2$	$-11x$	$+2$
x^2	$12x^4$	$-11x^3$	$2x^2$
x	$12x^3$	$-11x^2$	$2x$

$f(x)=(x+1)x(x-\frac{2}{3})(x-\frac{1}{4})\cdot 3\cdot 4$ any a

$f(x)=x(x+1)(3x-2)(4x-1)$

$=(x^2+x)(12x^2-11x+2)$

$\boxed{f(x)=12x^4+x^3-9x^2+2x}$

6. Write the polynomial function in standard form given the roots $x=3-2i$ and $x=1$. Also $x=3+2i$

$f(x)=(x-(3-2i))(x-(3+2i))(x-1)$

$=(x-3+2i)(x-3-2i)(x-1)$

$=(x^2-6x+13)(x-1)$

	x^2	$-6x$	$+13$
x	x^3	$-6x^2$	$13x$
-1	$-x^2$	$+6x$	-13

$\boxed{f(x)=x^3-7x^2+19x-13}$

7. Divide $x^3+4x^2-7x-10$ by $x+1$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -7 & -10 \\ & \downarrow & -1 & -3 & 10 \\ \hline & 1 & 3 & -10 & ☺ \end{array}$$

$\frac{x^3+4x^2-7x-10}{x+1}=\boxed{x^2+3x-10}$

8. Solve. $x^4-7x^2+12=0$

$(x^2-4)(x^2-3)=0$

$(x+2)(x-2)(x^2-3)=0$

$x^2-3=0$

$x^2=3$

$x=\pm\sqrt{3}$

$\boxed{x=-2,\ 2,\ \pm\sqrt{3}}$

9. Determine all the roots of $P(x)=x^4+x^2-14x-48$

$\frac{P}{Q}$ $\pm 1,\pm 2\ \pm 3\ \pm 4\ \pm 6\ \pm 8\ \pm 12\ \pm 16\ \pm 24\ \pm 48$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 1 & -14 & -48 \\ & \downarrow & 3 & 9 & 30 & 48 \\ \hline & 1 & 3 & 10 & 16 & ☺ \end{array}$$

$\boxed{x=3}$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & 10 & 16 \\ & \downarrow & -2 & -2 & -16 \\ \hline & 1 & 1 & 8 & ☺ \end{array}$$

$\boxed{x=-2}$

$x^2+x+8=0$ $x=\frac{-1\pm\sqrt{1-32}}{2}$

$\boxed{\frac{-1\pm i\sqrt{31}}{2}}$

10. Determine all the roots of $x^4+6x^3+10x^2+6x+9=0$

$\frac{P}{Q}=\pm 1\ \pm 3\ \pm 9$

$$\begin{array}{r|rrrrr} -1 & 1 & 6 & 10 & 6 & 9 \\ & \downarrow & -1 & -5 & -5 & -1 \\ \hline & 1 & 5 & 5 & 1 & 8 \end{array}$$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & \downarrow & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & ☺ \end{array}$$

$\boxed{x=-3}$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & \downarrow & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & ☺ \end{array}$$

$\boxed{x=-3}$

$x^2+1=0$

$x^2=-1$

$\boxed{x=\pm i}$

11. Determine the degree, zeros and their multiplicity.

$y = x^2(x+2)^3(x+1)$

Degree: 6

Zeros: $x = 0$ $\quad x = -2$ $\quad x = -1$

Multiplicity: 2 $\quad$ 3 $\quad$ 1

12. Solve. $\log_3(x) + \log_3(x+3) = \log_3 18$

$\log_3(x^2+3x) = \log_3 18$

$x^2 + 3x = 18$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$\cancel{x=-6}$

$\boxed{x = 3}$

13. Expand the logarithm. $\log\left(\frac{\sqrt[5]{2b^4}}{3x^3}\right)$ $\leftarrow 2^{\frac{1}{5}} b^{\frac{4}{5}}$ $\leftarrow 3^1 x^3$

$\boxed{\frac{1}{5}\log 2 + \frac{4}{5}\log b - \log 3 - 3\log x}$

14. Find an exponential growth model ($y = ab^x + k$) whose graph passes through the points (1,120) and (2, 168) and has the asymptote $y = 100$

$(1,120) \rightarrow 120 = a \cdot b^1 + 100 \rightarrow 20 = a \cdot b^1 \rightarrow a = \frac{20}{b}$

$(2,168) \rightarrow 168 = a \cdot b^2 + 100 \rightarrow 68 = a \cdot b^2$

$a = 20 \div b \rightarrow 20 \cdot \frac{20}{68}$ $\quad 68 = \frac{20}{b} \cdot b^2$

$a = \frac{400}{68}$ $\quad 68 = 20b$

$\boxed{y = \left(\frac{400}{68}\right)\left(\frac{68}{20}\right)^x + 100}$ $\quad b = \frac{68}{20}$

15. Find the inverse of $f(x) = \sqrt{2x+5} - 7$

$x = \sqrt{2y+5} - 7$

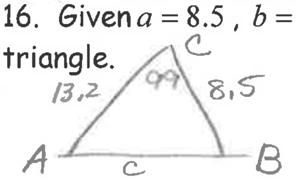
$x + 7 = \sqrt{2y+5}$

$(x+7)^2 = 2y + 5$

$(x+7)^2 - 5 = 2y$

$\boxed{y = \frac{(x+7)^2 - 5}{2}}$

16. Given $a = 8.5$, $b = 13.2$ and $C = 99°$, solve the triangle.



$c^2 = 13.2^2 + 8.5^2 - 2(13.2)(8.5)\cos 99$

$\boxed{c \approx 16.78}$

$\frac{\sin 99}{16.78076} = \frac{\sin B}{13.2}$

$\boxed{B \approx 50.98°}$

$A + B + C = 180$

$\boxed{A \approx 30.02°}$

17. Circle <u>all</u> of the following statements that apply to the graph of $f(x) = -3x(x-5)^3(x+2)^2$.

A. The degree is 6.
B. It has negative orientation.
C. It has 6 distinct x-intercepts.
D. The range is all real numbers.
E. It passes through the origin.
F. (0, 2) is one of the roots.
G. Its inverse is a function.
H. The y-intercept is (0, -500).

18. Find the inverse: $g(x) = 3\log_5(x+1) - 2$

$x = \log_5(y+1)^3 - 2$

$x + 2 = \log_5(y+1)^3$

$5^{x+2} = (y+1)^3$

$\sqrt[3]{5^{x+2}} = y + 1$

$\boxed{y = \sqrt[3]{5^{x+2}} - 1}$

19. Use synthetic division to show that 3i is a zero of $g(x) = x^3 + 4x^2 + 9x + 36$

3i	1	4	9	36
	↓	3i	12i−9	−36
	1	4+3i	12i	☺

since R=0

x=3i is a zero

20. Factor

a. $32x^{19} - 162x^3y^4$

$2x^3[16x^{16} - 81y^4]$ $\leftarrow (4x^8)^2 - (9y^2)^2$

$2x^3(4x^8 - 9y^2)(4x^8 + 9y^2)$

$= \boxed{2x^3(2x^4 - 3y)(2x^4 + 3y)(4x^8 + 9y^2)}$

b. $54x^6 - 16y^9$

$= 2(27x^6 - 8y^9) \leftarrow (3x^2)^3 - (2y^3)^3$

$= \boxed{2(3x^2 - 2y^3)(9x^4 + 6x^2y^3 + 4y^6)}$