**Trig Identities**

**1.** If A is an obtuse angle in a triangle and sin A = , calculate the exact value of sin 2A.

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| *Working:* |  |
|  | *Answer:*  ...................................................................... |

(Total 4 marks)

**2.** (a) Express 2 cos2 *x* + sin *x* in terms of sin *x* only.

(b) Solve the equation 2 cos2 *x* + sin *x* = 2 for *x* in the interval 0  *x*  , giving your answers exactly.

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| *Working:* |  |
|  | *Answers*:  (a) ..................................................................  (b) .................................................................. |

(Total 4 marks)

**3.** (a) Write the expression 3 sin2 *x* + 4 cos *x* in the form *a* cos2 *x* + *b* cos *x* + *c*.

(b) Hence or otherwise, solve the equation

3 sin2 *x* + 4 cos *x* – 4 = 0, 0  *x*  90.

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| *Working:* |  |
|  | *Answers*:  (a) ..................................................................  (b) .................................................................. |

(Total 4 marks)

**4.** Consider the trigonometric equation 2 sin2 *x* = 1 + cos *x*.

(a) Write this equation in the form *f* (*x*) = 0, where *f* (*x*) = *a* cos2 *x* + *b* cos *x* + *c*,  
and *a*, *b*, *c*  .

(b) Factorize *f* (*x*).

(c) Solve *f* (*x*) = 0 for 0°  *x*  360°.

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| *Working:* |  |
|  | *Answers*:  (a) ..................................................................  (b) ..................................................................  (c) .................................................................. |

(Total 6 marks)

**5.** Solve the equation 2 cos2 *x* = sin 2*x* for 0  *x*  *π*, giving your answers in terms of *π*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:  ………………………………………….. |

(Total 6 marks)

**6.** The function *f* is defined by *f* : *x*  30 sin 3*x* cos 3*x*, 0  *x*  .

(a) Write down an expression for *f* (*x*) in the form *a* sin 6*x*, where *a* is an integer.

(b) Solve *f* (*x*) = 0, giving your answers in terms of .

(Total 6 marks)

**7.** Given that sin *x* = , where *x* is an acute angle, find the **exact** value of

(a) cos *x*;

(b) cos 2*x*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:  (a) ..................................................................  (b) .................................................................. |

(Total 6 marks)

**8.** Let *f* (*x*) = *a* (*x* − 4)2 + 8.

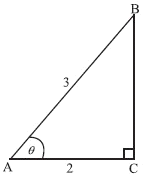
(a) Write down the coordinates of the vertex of the curve of *f*.

(b) Given that *f* (7) = −10, find the value of *a*.

(c) Hence find the *y*-intercept of the curve of *f*.

(Total 6 marks)

**9.** The following diagram shows a triangle ABC, where  is 90, AB = 3, AC = 2 and  is **.



(a) Show that sin ** = .

(b) Show that sin 2** = .

(c) Find the **exact** value of cos 2*.*

(Total 6 marks)

**10.** Solve the equation 2cos *x* = sin 2*x*, for 0 ≤ *x* ≤ 3π.

(Total 7 marks)

**11.** The straight line with equation *y* =  makes an acute angle *θ* with the *x*-axis.

(a) Write down the value of tan *θ*.

(1)

(b) Find the value of

(i) sin 2*θ*;

(ii) cos 2*θ*.

(6)

(Total 7 marks)

**12.** (a) Show that 4 – cos 2*θ* + 5 sin *θ* = 2 sin2 *θ* + 5 sin *θ* + 3.

(2)

(b) **Hence**, solve the equation 4 – cos 2*θ* + 5 sin *θ* = 0 for 0 ≤ *θ* ≤ 2π.

(5)

(Total 7 marks)

**13.** Let *f*(*x*) = sin3 *x* + cos3 *x* tan *x*,  < *x* < π*.*

(a) Show that *f*(*x*) = sin *x*.

(2)

(b) Let sin *x* = . Show that *f*(2*x*) = .

(5)

(Total 7 marks)

**14.** Solve cos 2*x* – 3 cos *x* – 3 – cos2 *x* = sin2 *x*, for 0 ≤ *x* ≤ 2π.

(Total 7 marks)

**15.** (a) Consider the equation 4*x*2 + *kx* + 1 = 0. For what values of *k* does this equation have two **equal** roots?

(3)

16. Let *f* be the function *f* (** ) = 2 cos 2** + 4 cos ** + 3, for −360  **  360.

(b) Show that this function may be written as *f* (** ) = 4 cos2 ** + 4 cos ** + 1.

(1)

(c) Consider the equation *f* (** ) = 0, for −360  **  360.

(i) How many distinct values of cos ** satisfy this equation?

(ii) Find all values of ** which satisfy this equation.

(5)

(d) Given that *f* (** ) = *c* is satisfied by only three values of **, find the value of *c*.

(2)

(Total 11 marks)